Lecture 14:

Image blur model: $g(x,y) = h * f(x,y) + n(x,y)$

In the frequency domain,

$$G(u,v) = c\,H(u,v)\,F(u,v) + N(u,v)$$

c constant

$\therefore$ Deblurring can be done by:

Compute: $F(u,v) \approx \dfrac{G(u,v)}{c\,H(u,v)}$

$G(u,v)$ — from observed image

$c\,H(u,v)$ — from known degradation

Obtain: $f(x,y) = DFT^{-1}(F(u,v))$

(Does NOT work very well due to noise!)

Recall:

Image deblurring in the frequency domain: (Assume H is known)

Method 1: Direct inverse filtering

Let $T(u,v) = \dfrac{1}{H(u,v) + \underbrace{\varepsilon\, \mathrm{sgn}(H(u,v))}_{\text{Avoid singularity}}}$ ($\mathrm{sgn}(z) = 1$ if $\mathrm{Re}(z) \geq 0$ and $\mathrm{sgn}(z) = -1$ otherwise)

Compute $\hat{F}(u,v) = G(u,v)\, T(u,v)$.

Find inverse DFT of $\hat{F}(u,v)$ to get an image $\hat{f}(x,y)$.

Good: Simple

Bad: Boast up noise

$$\hat{F}(u,v) = \underbrace{G(u,v)}_{= H(u,v)F(u,v) + N(u,v)} T(u,v) \approx F(u,v) + \frac{N(u,v)}{H(u,v) + \varepsilon\, \mathrm{sgn}(H(u,v))}$$

Note: $H(u,v)$ is big for (u,v) close to $(0,0)$ (Keep low frequencies)

is small for (u,v) far away from $(0,0)$

$\therefore \dfrac{N(u,v)}{H(u,v) + \varepsilon\, \mathrm{sgn}(H(u,v))}$ is big for (u,v) far away from $(0,0)$

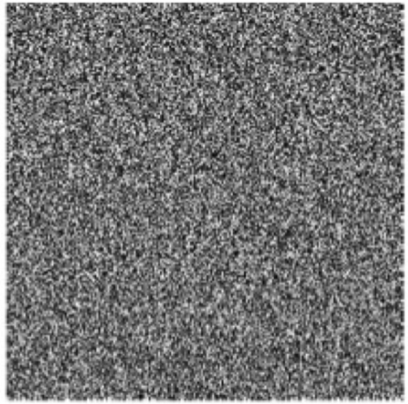
Large gain in high frequencies

$\Downarrow$

Boast up noises!!

Original

Blurred image



Direct inverse filtering

Recall:

Method 2: Modified inverse filtering

Let $B(u,v) = \dfrac{1}{1+\left(\dfrac{u^2+v^2}{D^2}\right)^n}$ and $T(u,v) = \dfrac{B(u,v)}{H(u,v)+\varepsilon \operatorname{sgn}(H(u,v))}$.

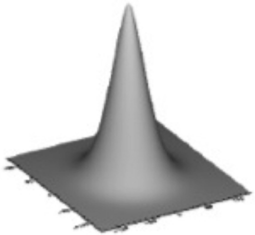
Then define: $\hat{F}(u,v) = T(u,v)\,G(u,v) \approx F(u,v)\,B(u,v) + \dfrac{N(u,v)\,B(u,v)}{H(u,v)+\varepsilon \operatorname{sgn}(H(u,v))}$

$\dfrac{N(u,v)\,B(u,v)}{H(u,v)+\varepsilon \operatorname{sgn}(H(u,v))} \approx \dfrac{N(u,v)}{H(u,v)+\varepsilon \operatorname{sgn}(H(u,v))}$ for $(u,v) \approx (0,0)$

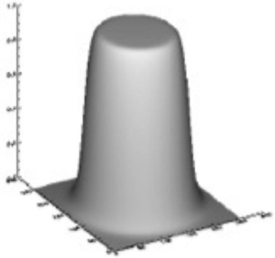
$\dfrac{N(u,v)\,B(u,v)}{H(u,v)+\varepsilon \operatorname{sgn}(H(u,v))}$ is small (as $B(u,v)$ is small) for (u,v) far away from $(0,0)$.

$\dfrac{B(u,v)}{H(u,v)+\varepsilon \operatorname{sgn}(H(u,v))}$ suppresses the high-frequency gain.

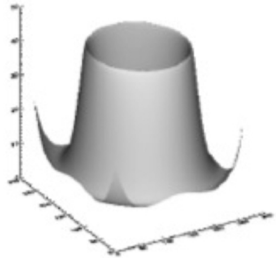
Bad: Has to choose D and n carefully.



$H(u,v)$



$B(u,v)$: $\mathsf{D} = 90, n = 8$



Inverse B/H

Original Image $G(u,v)$

Blurred using $\mathsf{D} = 90, n = 8$

Restored with a best D and n.

Method 3: Wiener filter

Let $T(u,v) = \dfrac{\overline{H(u,v)}}{|H(u,v)|^2 + \frac{S_n(u,v)}{S_f(u,v)}}$ where $S_n(u,v) = |N(u,v)|^2$

$S_f(u,v) = |F(u,v)|^2$

If $S_n(u,v)$ and $S_f(u,v)$ are not known, then we let $K = \dfrac{S_n(u,v)}{S_f(u,v)}$ to get:

$$T(u,v) = \frac{\overline{H(u,v)}}{|H(u,v)|^2 + K}$$

Let $\hat{F}(u,v) = T(u,v)\,G(u,v)$. Compute $\hat{f}(x,y) =$ inverse DFT of $\hat{F}(u,v)$.

In fact, the Wiener filter can be described as an inverse filtering as follows:

$$\hat{F}(u,v) = \left[\left(\frac{1}{H(u,v)}\right)\left(\frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right)\right] G(u,v)$$

Behave like "Modified inverse filtering"

≈ 0 if $H(u,v) \approx 0$ (if (u,v) far away from 0)

≈ 1 if $H(u,v)$ is large (if $(u,v) \approx (0,0)$)

What does Wiener filtering do mathematically?

We can show: Wiener filter minimizes the mean square error:

$$\underbrace{\varepsilon^2(f, \hat{f})}_{} = \sum_{x=-\frac{N}{2}}^{\frac{N}{2}} \sum_{y=-\frac{N}{2}}^{\frac{N}{2}} \left| f(x,y) - \hat{f}(x,y) \right|^2$$

original Restored

degradation

Observed

$$\text{Let } g = h * f + n$$

noise

original

Then, the restored image $\hat{f}(x,y)$ can be written as:

$$\hat{f}(x,y) = w(x,y) * g(x,y) \text{ for some } w(x,y)$$

Recall: $\hat{f}$ is obtained as follows:

Step 1: Let $\hat{F}(u,v) = \underbrace{W(u,v)}_{\text{Filter}} G(u,v)$ $\quad (G(u,v) = DFT(g)(u,v))$

Step 2: Compute iFT of $\hat{F}$ to get $\hat{f}$

$\therefore \hat{f} = w * g$ for some w. $\quad \left(\text{or } \hat{F} = DFT(\hat{f}) = W \odot G \right)$

Thus, $\hat{f}$ denpends on W

We can regard $\mathcal{E}^2(\hat{f}, f)$ as a functional depending on W:

$$\mathcal{E}^2(\hat{f}, f) = \mathcal{E}^2(W)$$

We can find an optimal W that minimizes:

$$\mathcal{E}^2(W) = \| \hat{f}(W) - f \|_F^2$$

Under suitable condition (spatially correlated), the minimizer W is given by:

$$W(u,v) = \frac{\overline{H(u,v)}}{|H(u,v)|^2 + \frac{S_n(u,v)}{S_f(u,v)}}$$

where $S_n(u,v) = |N(u,v)|^2$

$S_f(u,v) = |F(u,v)|^2$